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Motion in Paraboloidal Coordinates:

Applications to $J2$ Gravity Perturbed Trajectories of Space Dynamics

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Abstract

In this paper, initial value problem for dynamical astronomy will be established using paraboloidal coordinates. A computational algorithm is developed for the initial value problem of $J2$ ***gravity perturbed trajectories of the space dynamics. Applications of the algorithm for the problem of final state prediction are illustrated by numerical examples of some test orbits of different eccentricities. The numerical results are extremely accurate and efficient in predicting final state for*** $J2$ ***gravity perturbed trajectories which is important for scientific researches. Moreover, an additional efficiency of the algorithm is that, for each of the test orbits, the step size used for solving the differential equations of motion is at most 70% of the step size used for obtaining its reference final state solution.***

Keywords: Dynamical astronomy**,** stellar dynamics, orbit determination, initial value problems**.**

1 Introduction

Depending on the application, a curvilinear coordinate system in general, may be simpler to use than the Cartesian coordinate system. For instance, in the galactic rotation, cylindrical coordinates are usually adopted, while the spherical coordinates are suitable for the dynamics of globular clusters.

The application of the conventional equations of space dynamic for the motion of Earth's artificial satellites gives inaccurate prediction for their positions and velocities. This is because these equations are unstable in the Liapunov sense [12]. In brief, the deficiency of these equations is due to the choice of the variables, which in turn has led some authors to propose successful devices to change of the dependent and /or independent variables so as to regularize the differential equations of motion.

Of these, the method established by Stiefel and Scheifele, in 1971.This method consists of changing the independent variable from time to a new variable, which is proportional to the eccentric anomaly in the elliptic case or its equivalent in hyperbolic case. The method is then changes the coordinates from three-dimensional Cartesian space to a four –dimensional space by what they called the KS transformation. The resulting equations are four –dimensional harmonic oscillator.

In fact, the change of the dependent and/or independent variables for the differential equations of motion is one of the focal points of researches in space dynamics. Many studies on the applications of these devices for some orbital systems were done [1-11] .The results of these applications were wonderful in obtaining accurate predictions of the final state which is of extreme importance for scientific researches as well as for military purposes.

Now, one may ask: Is there exist another transformation equations (other than mentioned above) that produce accurate final state predication? The answer is the present paper which produces upon using paraboloidal coordinates, very accurate final state predictions, as judged by the error criteria ΔR (for the final position)and Δv (for the final velocity). Moreover, an additional efficiency of using paraboloidal coordinates is that, one can reach the desired accuracy using at most 70% of the number of steps that used for obtaining the reference final state solution. By this reduction, the step size becomes larger, hence minimizing the computational errors. Finally it should be mentioned that , by using paraboloidal coordinates the independent variables are only, changed which in turn produce transformations from three dimensional Cartesian space to another three dimensional space.

In the present paper, initial value problem for dynamical astronomy will be established using paraboloidal coordinates. Computation algorithm was developed for the initial value problem of $J2$ gravity perturbed trajectories. Applications of the algorithm for the problem of final state predication are illustrated by numerical examples of some test orbits of different eccentricities.

2 Analytical Formulations for Paraboloidal Coordinates

2.1 Coordinate, velocity transformations and the scale factors

$$x = u_1 u_2 \cos u_3 \quad ; \qquad y = u_1 u_2 \sin u_3 \quad ; \qquad z = \frac{1}{2}(u_1^2 - u_2^2) \ , \tag{1}$$

$$\dot{x} = u_2\dot{u}_1 \cos u_3 + u_1\dot{u}_2 \cos u_3 - u_1u_2\dot{u}_3 \sin u_3;$$

$$\dot{y} = u_2\dot{u}_1 \sin u_3 + u_1\dot{u}_2 \sin u_3 + u_1u_2\dot{u}_3 \cos u_3; \quad (2)$$

$$\dot{z} = u_1\dot{u}_1 - u_2\dot{u}_2.$$

where

$$0 \le u_1 < \infty, \quad 0 \le u_2 < \infty, \quad -\pi \le u_3 \le \pi$$

The scale factors of the transformation are

$$h_1^2 = h_2^2 = u_1^2 + u_2^2 \quad , \quad h_3^2 = u_1^2 u_2^2$$

2.2 Inverse transformations

Since

$$r = (x^2 + y^2 + z^2)^{1/2} \quad (3)$$

then we get from Equation (1) that:

$$u_1^2 + u_2^2 = 2r. \quad (4)$$

Since u_1 and u_2 are both non-negative, then we get from Equations (1) and (4) that

$$u_1 = (r+z)^{1/2} \quad ; \quad u_2 = (r-z)^{1/2} \quad ; \quad u_3 = \arctan(\frac{y}{x}) \quad (5)$$

Differentiating the last of Equations (1) and Equations (3) and (4) with respective to t we get:

$$\dot{u}_1 = \frac{[x\dot{x} + y\dot{y} + \dot{z}(z+r)]}{2ru_1} \quad ; \quad \dot{u}_2 = \frac{[x\dot{x} + y\dot{y} + \dot{z}(z-r)]}{2ru_2} \quad ; \quad \dot{u}_3 = \frac{x\dot{y} - y\dot{x}}{x^2 + y^2} \quad (6)$$

where *r* (hence u_1 and u_2)is given in terms of (x, y, z) from Equation (3).

2.3 Equations of motion

In the present paper we shall suppose that the motion is controlled by a gravitational potential V, which will be in general a function of (u_1, u_2, u_3).

From the above equations we get after some calculations that :

$$\dot{u}_1 = u_4$$

$$\dot{u}_2 = u_5$$

$$\dot{u}_3 = u_6$$

$$\dot{u}_4 = \frac{1}{u_1^2 + u_2^2}\{-u_4^2 u_1 + u_1 u_5^2 - 2u_2 u_4 u_5 + u_1 u_2^2 u_6^2 + \frac{\partial V}{\partial u_1}\}$$

$$\dot{u}_5 = \frac{1}{u_1^2 + u_2^2}\{u_4^2 u_2 - u_2 u_5^2 - 2u_1 u_4 u_5 + u_2 u_1^2 u_6^2 + \frac{\partial V}{\partial u_2}\} \ ;$$

$$\dot{u}_6 = -\frac{2u_6 u_4}{u_1} - \frac{2u_6 u_5}{u_2} + \frac{1}{u_1^2 u_2^2}\frac{\partial V}{\partial u_3} \ .$$

It should be noted that the equations of the present section are general in the sense that it could be applied for any dynamical system.

3 J2 gravity perturbed trajectories

3. 1 The potential V and its partial derivatives

For J2 gravity perturbed trajectories, the potential V is given as:

$$V \equiv V(x,y,z) = \frac{\mu}{r} + \frac{c}{r^3}[3\left(\frac{z}{r}\right)^2 - 1] \qquad (7)$$

where

$$c = J_2 \mu R_\otimes^2 / 2 \quad ; \ r = (x^2 + y^2 + z^2)^{1/2},$$

with μ is the gravitational parameter ,which is universal gravitational constant times the

Earth's mass ; J_2 the second zonal harmonic ,and $R_{\oplus}$ is the mean Earth's equatorial radius .The numerical values of these constants are:

$$\mu = 398600.8 \, \mathrm{km^3 s^{-2}} \,,$$

$$J_2 = 1.0826157 \times 10^{-3} \,,$$

$$R_{\otimes} = 6378.135 \ \mathrm{km}.$$

From Equation (7) we have:

$$\frac{\partial V}{\partial x} = -\frac{\mu x}{r^3} + 3c\left(\frac{x}{r^5}\right)\left\{1 - \frac{5z^2}{r^2}\right\} \,, \tag{8.1}$$

$$\frac{\partial V}{\partial y} = -\frac{\mu y}{r^3} + 3c\left(\frac{y}{r^5}\right)\left\{1 - \frac{5z^2}{r^2}\right\} , \tag{8.2}$$

$$\frac{\partial V}{\partial z} = -\frac{\mu z}{r^3} + 3c\left(\frac{z}{r^5}\right)\left\{3 - \frac{5z^2}{r^2}\right\} \ . \tag{8.3}$$

3.2 Initial value algorithm

In what follows, the initial value algorithm for J2 gravity perturbed trajectories in paraboloidal coordinates will be considered. The algorithm is described through its basic points: input, output and computational steps

Input: (1) $x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0$ at $t = t_0$**,**

(2) The flight time $t = t_f$ **;**

(3) $\frac{\partial V}{\partial x}$; $\frac{\partial V}{\partial y}$ and $\frac{\partial V}{\partial z}$; (Equations (8))

Output: $x, y, z; \dot{x}, \dot{y}, \dot{z}$ at $t = t_f$

Computational steps:

1- Using Equations (1) and (8) to find the analytical expressions of the partial derivatives $\frac{\partial V}{\partial u_1};\frac{\partial V}{\partial u_2};\frac{\partial V}{\partial u}$ as functions in $u_j; j=1,2,3$ as:

$$\frac{\partial V}{\partial u_1}=-\frac{4u_1\left(u_1^2+u_2^2\right)\left(\mu u_1^8+4\mu u_1^6 u_2^2+72c\,u_2^4+\mu u_2^8+4u_1^2 u_2^2\left(-36c+\mu u_2^4\right)+6u_1^2\left(4c+\mu u_2^4\right)\right)}{\left(\left(u_1^2+u_2^2\right)^2\right)^{7/2}}$$

$$\frac{\partial V}{\partial u_2}=-\frac{4u_2\left(u_1^2+u_2^2\right)\left(72c\,u_1^4+\mu u_1^8+4u_1^2\left(-36c+\mu u_1^4\right)u_2^2+6\left(4c+\mu u_1^4\right)u_2^4+4\mu u_1^2 u_2^6+\mu u_2^8\right)}{\left(\left(u_1^2+u_2^2\right)^2\right)^{7/5}}$$

$$\frac{\partial V}{\partial u_3}=0$$

2-Compute the initial conditions, $u_{0j}; j=1,2,\cdots,6$ for the differential system applying the transformations: $(x,y,z)\to(x_0,y_0,z_0)$ and $(\dot{x},\dot{y},\dot{z})\to(\dot{x}_0,\dot{y}_0,\dot{z}_0)$

3- Using these initial conditions to solve numerically the differential system for u_j; $j=1,2\cdots,6$ at $t=t_f$,where $u_4\equiv\dot{u}_1, u_5\equiv\dot{u}_2, u_6\equiv\dot{u}_3$ at $t=t_f$

4- Using $u_j;\dot{u}_j; j=1,2,3$ to compute x,y,z and $\dot{x},\dot{y},\dot{z}$ at $t=t_f$ from the direct transformations of Equations (1)

5-End

3.3 Numerical applications

The purpose of this section is to demonstrate the efficiency of the initial value problem using paraboloidal coordinates in producing accurate final state predictions for J2 gravity perturbed trajectories

3.3.1 Test orbits

For the applications of the above formulations, we consider some test orbits given in the Appendix C of Vinti's book [13]. All these orbits have the initial time $t_0 = 0$ and each of different flight time t_f, they cover the three basic types of conic motion-elliptic, parabolic and hyperbolic orbits characterized by the initial conditions listed together with t_f, in the first columns of the tables of Appendix A of the present paper. The components of the position vector for each orbit are in km, while the corresponding components of the velocity vector are in $\mathrm{km}\, s^{-1}$.

3.3.2 Reference orbits

For each orbit, the J2 gravity perturbed equations of motion in Cartesian coordinate are solved by the classical Runge-Kutta integrator. A final state prediction was determined by reducing the step size until at least five decimal places $(\prec 10^{-2}$ meter (m)) stabilized in $x(t_f), y(t_f)$ and $z(t_f)$. These values are considered as reference final states solutions to the orbit they refer and are denoted by :

$$\mathbf{r}_R \equiv (x_R(t_f), y_R(t_f), z_R(t_f)) \quad \text{and} \quad \dot{\mathbf{r}}_R \equiv (\dot{x}_R(t_f), \dot{y}_R(t_f), \dot{z}_R(t_f)) \tag{9}$$

for the reference position and velocity vectors respectively. The components of these vectors are listed for each orbit in the second columns of the tables of Appendix A.

3.3.3 Efficiency of cylindrical polar coordinates

Upon the above reference solutions the efficiency of the initial value problem for J2 gravity perturbed trajectories using paraboloidal coordinates (PC- solution) may be checked by testing its ability in predicting final states within certain tolerances as follows Let $\mathbf{r} \equiv (x\ (t_f), y(t_f), z\ (t_f))$ and $\dot{\mathbf{r}} \equiv (\dot{x}\ (t_f), \dot{y}(t_f), \dot{z}\ (t_f))$ are the final state of the

PC- solution of a given orbit .The efficiency of the PC- solution are then checked by the magnitude of the error criteria ΔR and Δv as:

$$\Delta R = \left\{(x - x_R)^2 + (y - y_R)^2 + (z - z_R)^2\right\}^{1/2} \times 1000 \quad \text{(in m)}, \tag{10}$$

$$\Delta v = \left\{(\dot{x} - \dot{x}_R)^2 + (\dot{y} - \dot{y}_R)^2 + (\dot{z} - \dot{z}_R)^2\right\}^{1/2} \times 1000 \quad \text{(in m s}^{-1}\text{)}, \tag{11}$$

such that ,the small values of ΔR and Δv ,the higher efficiency will be, in this respect , we may define an acceptable solution set (**S.S)** to the problem at hand as:

$$\mathbf{S.S} = \left((\mathbf{r}, \dot{\mathbf{r}}) \mathbf{:} \, \Delta R \le \varepsilon_1 , \Delta v \le \varepsilon_2 \right) \tag{11}$$

where $\varepsilon_{1,2}$ are given tolerances . For the very accurate predictions required nowadays we may consider the tolerances $\varepsilon_{1,2}$ as:

$$\varepsilon_1 = 1 \text{ meter} \pm 10 \text{ centimeter}, \tag{12.1}$$

$$\varepsilon_2 = .25 \text{ m s}^{-1}. \tag{12.2}$$

The components of the position and velocity vectors $\mathbf{r}$ and $\dot{\mathbf{r}}$ of the PC solution are listed for each of the test orbits in the third columns of the tables of Appendix A., while the values of the errors ΔR and Δv of Equations (10) and(11) are given at the bottom of each table.

These values indicated in accordance of the acceptance solution set that, the PC solution is very accurate and efficient in predicating final state for J2 gravity perturbed trajectories which is importance for scientific researches. Moreover, the step size used in the differential solver for obtaining the PC solution for each of the test orbit is at most 70% of the step size used for obtaining the reference solution.

Conclusion

The results of these applications are wonderful in obtaining very accurate predictions of the final state which is extremely importance for scientific researches**.** By, using our new method, we can reach the accuracy of one cm. using about 70% of the number of steps that used for obtaining its reference final state solution. By this reduction, the step size becomes larger, hence minimizing the computational errors

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Appendix A: Numerical Results

Low-Earth Orbit Initial conditions	Reference Solution	PC-Solution
X_o=2328.9694	X_R=-516.450939	X=-516.455092
Y_o=-5995.21600	Y_R=-3026.5115474	Y=-3026.511568
Z_o=1719.97894	z_R=5848.117544	Z=5848.117534
$X^{\bullet}_{\circ}=2.911101130$	$X^{\bullet}_{R}=3.96699$	$X^{\bullet}=3.966599$
$Y^{\bullet}_{\circ}=-0.98164053$	$Y^{\bullet}_{R}=-6.121618$	$Y^{\bullet}=-6.121618$
$Z^{\bullet}_{\circ}=-.090499220$	$Z^{\bullet}_{R}=-2.754866$	$Z^{\bullet}_{R}=-2.754866$

t_f=10 000 sec

ΔR =0.00602(m) ΔV=0.0 (m/sec)

Molniya Orbit

Initial conditions	Reference Solution	PC-Solution
X_o=19850.34032	X_R=19868.05645	X=19868.06535
Y_o=-40076.98310	Y_R=-39990.912262	Y=-9990.912261
Z_o=5686.51314	z_R=5800.4214132	Z=5800.4214082
$X^{\bullet}_{\circ} = 0.9622473922$	$X^{\bullet}_{R} = 0.970945$	$X^{\bullet} = 0.970945$
$Y^{\bullet}_{\circ} = -0.3840200243$	$Y^{\bullet}_{R} = -0.397016$	$Y^{\bullet} = -0.397016$
$Z^{\bullet}_{\circ} = -1.2806877932$	$Z^{\bullet}_{R} = -1.278460$	$Z^{\bullet}_{R} = -1.278460$

t_f=86 400 sec

ΔR =0.00726(m) ΔV=0.0×10^{-6}(m/sec)

Geosynchronous Orbit

Initial conditions	Reference Solution	PC-Solution
X_o=-14420.99601	X_R=-13755.532790	X=-13755.532790
Y_o=-39621.36091	Y_R=-39857.2791670	Y=-39857.279167
Z_o=0	z_R=0	Z=0
$X^{\bullet}_{\circ} = 2{,}88923555010$	$X^{\bullet}_{R} = 2.906438$	$X^{\bullet} = 2.906438$
$Y^{\bullet}_{\circ} = -1.0515957400$	$Y^{\bullet}_{R} = -1.003071$	$Y^{\bullet} = -1.003071$
$Z^{\bullet}_{\circ} = 0$	$Z^{\bullet}_{R} = 0$	$Z^{\bullet}_{R} = 0$

t_f=86 400 sec

ΔR =0.000103817(m) ΔV=0.0 (m/sec)

Parabolic Orbit of Zero Inclination

Initial conditions	Reference Solution	PC-Solution
X_o=10 000.00	X_R=-65357.0633677	X=-65357.063369
Y_o=0	Y_R=54991.369699	Y=54991.369701
Z_o=0	z_R=0	Z=0
$X^{\bullet}_{\circ} = 0$	$X^{\bullet}_{R} = -2.871888$	$X^{\bullet} = -2.871888$
$Y^{\bullet}_{\circ} = 8.9286113142$	$Y^{\bullet}_{R} = 1.050276$	$Y^{\bullet} = 1.050276$
$Z^{\bullet}_{\circ} = 0$	$Z^{\bullet}_{R} = 0$	$Z^{\bullet}_{R} = 0$

t_f=21 600 sec

ΔR =0.00189(m) ΔV=0.0 (m/sec)

Hyperbolic Orbit of Zero Inclination

Initial conditions	Reference Solution	PC-Solution
$X_o=2328.96594$	$X_R=-1.898682002201\times10^6$	$X=-1.898682002277\times10^6$
$Y_o=0$	$Y_R=1.020564164530\times10^6$	$Y=1.020564164440\times10^6$
$Z_o=0$	$z_R=0$	$Z=0$
$X^{\bullet}_{\circ}=0$	$X^{\bullet}_{R}=-2.049040$	$X^{\bullet}=-2.049040$
$Y^{\bullet}_{\circ}=-0.98164053$	$Y^{\bullet}_{R}=1.052929$	$Y^{\bullet}=1.052929$
$Z^{\bullet}_{\circ}=0$	$Z^{\bullet}_{R}=0$	$Z^{\bullet}_{R}=0$

$t_f=10\ 000$ sec

$\Delta R=0.11831$(m) $\Delta V=0.0$ (m/sec)

Hyperbolic Orbit of 90° Inclination

Initial conditions	Reference Solution	PC-Solution
$X_o=10\ 000.0$	$X_R=179.642069$	$X=179.642069$
$Y_o=0$	$Y_R=0$	$Y=0$
$Z_o=0$	$z_R=0$	$Z=0$
$X^{\bullet}_{\circ}=0$	$X^{\bullet}_{R}=-4.332755$	$X^{\bullet}=-4.332755$
$Y^{\bullet}_{\circ}=0$	$Y^{\bullet}_{R}=0$	$Y^{\bullet}=0$
$Z^{\bullet}_{\circ}=9.2$	$Z^{\bullet}_{R}=4.905574600$	$Z^{\bullet}_{R}=4.90557460$

$t_f=3000$ sec

$\Delta R=0.00003$(m) $\Delta V=0.0$ (m/sec)

Exo-Atmospheric Interceptor Trajectory

Initial conditions	Reference Solution	PC-Solution
$X_o=-1221.14362$	$X_R=-1210.2567225$	$X=-1210.256722$
$Y_o=288.41648$	$Y_R=5274.987181$	$Y=5274.987181$
$Z_o=3502.50807$	$Z_R=3563.890150$	$Z=3563.890150$
$X^{\bullet}_{\circ}=0.0192755409$	$X^{\bullet}_{R}=0.1197914$	$X^{\bullet}=0.197914$
$Y^{\bullet}_{\circ}=0.25453560030$	$Y^{\bullet}_{R}=-0.521570$	$Y^{\bullet}=-0.521570$
$Z^{\bullet}_{\circ}=0.8722443619$	$Z^{\bullet}_{R}=0.354700$	$Z^{\bullet}_{R}=0.354711$

$t_f=100$ sec

$\Delta R=0.0$(m) $\Delta V=0.0$ (m/sec)

Long-Rang Ballistic
Missile Trajectory

Initial conditions	Reference Solution	PC-Solution
X_o=-3158.0000	X_R=-6474.1747537	X=-6474.1747537
Y_o=-4647.0000	Y_R=-3206.17473396	Y=-3206.687340
Z_o=3568.0000	z_R=1079.408375	Z=1079.408375
$X_{\circ}^{\bullet} = -5.7450000$	$X_{R}^{\bullet} = -0.529512$	$X^{\bullet} = -0.529512$
$Y_{\circ}^{\bullet} = -0.9720000$	$Y_{R}^{\bullet} = 3.387221$	$Y^{\bullet} = 3.387221$
$Z_{\circ}^{\bullet} = -0.8950000$	$Z_{R}^{\bullet} = -3.509546$	$Z_{R}^{\bullet} = -3.509546$
t_f=1000 sec		
ΔR =0.0 (m)	ΔV=0.0 (m/sec)	