

Effect of Linear Threshold Gate on the Countenance of Artificial Neural Network

Arunachalam Solairaju¹, Rajupillai Periyasamy²,
¹Jamal Mohamed College, Tiruchirappalli, India,
²Nehru Memorial College, Puthanampatti, India,

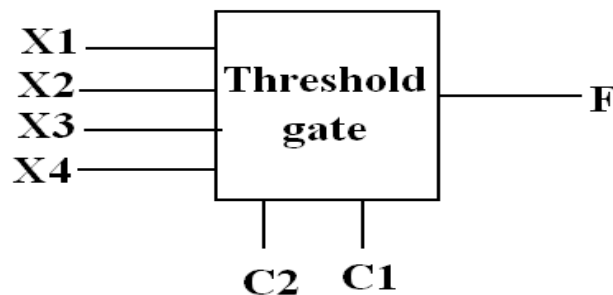
Abstract: Human brain is far superior to computers in solving hard problems like combinatorial optimization and image and speech recognition, although its basic building blocks are several orders of magnitude slower. This observation has boosted interest in the field of artificial neural network. We consider the Boolean version of an artificial neuron namely a linear threshold element which computes a neural like Boolean function of n binary inputs. The main issues in the study of LT element called LT circuits, include the estimation of the computational capabilities and the comparison of their properties with the existing AND, OR and NOT gates (called AON circuits). There is strong evidence that LT circuits are more efficient than AON circuits in implementing a number of important function including addition, multiplication and division of integer. In this paper we discuss that a single n input Linear Threshold Gate is a much more Powerful gate than a single n-input NAND or NOR gate.

Introduction: Artificial neural networks are viable computational models for a wide variety of problems. These include pattern classification, speech synthesis and recognition, and etc. These networks have been inspired by neuroscience. This paper investigates the computational capability of a linear threshold gate (LTG).

2. Threshold Gates: A threshold gate is one in which at least T inputs must be logic 1 in order to produce a logic 1 at the gate output. T is an integer value representing the threshold of the gate. If X_1, X_2, \dots, X_n are the inputs, F is the gate output and T is the threshold then the operation can be described by the following functions.

$$F = 1 \text{ if } X_1 + X_2 + \dots + X_n \geq T ; F = 0 \text{ if } X_1 + X_2 + \dots + X_n < T.$$

The figure bellow shows a reconfigurable threshold gate with a total of six inputs. X_1, X_2, X_3 and X_4 are the input signals C_2 & C_1 are configuration signals.



If $C_2C_1 = 11$ then the gate functions as an OR gate. If $C_2C_1 = 00$ then the gate functions as an AND gate. Other threshold functions exists between these two sceneries. In general the effective threshold seen by the gate input signal is $4 - (2C_2 + C_1)$. $F = 1$ if $X_1 + X_2 + X_3 + X_4 \geq 4 - (2C_2 + C_1)$; $F = 0$ if $X_1 + X_2 + X_3 + X_4 < 4 - (2C_2 + C_1)$

2.1 Linear Threshold Gates:-

The basic function of a linear threshold gate is to discriminate between labeled points. An LTG maps a vector of input data x and output y . The transfer function is given by

$$y = \begin{cases} 1 & \text{if } W^T X = \sum_{i=1}^n w_i x_i \geq T \\ 0 & \end{cases}$$

where $X = [X_1, X_2, \dots, X_n]$.

$W = [W_1, W_2, \dots, W_n]$ are the input and weight respectively. T is a threshold constant. The symbolic representation of an LTG is as shown in the figure 1 and the graphical representation as shown in the fig 2.

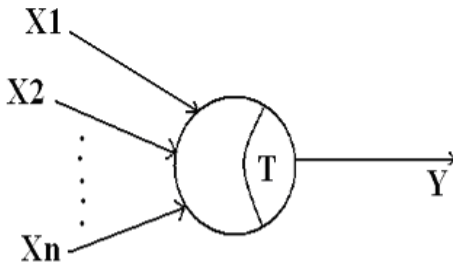


Figure 1

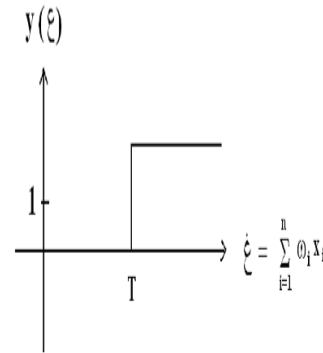


Figure 2

LTG output may assume either of the following mapping forms

$$X \in \{0,1\} \Rightarrow y : \{0,1\}^n \longrightarrow \{0,1\}$$

(or)

$$X \in \mathbf{R}^n \Rightarrow y : \mathbf{R}^n \longrightarrow \{0,1\}$$

2.2 Boolean function realization using LTG:

$$Y = (X_1, X_2, X_3) = \bar{X}_1 X_2 + \bar{X}_2 X_3$$

The weight vector $w = [-1 \ 2 \ -1]$ and threshold $T = \frac{1}{2}$ there are 8 inequalities. They are

1) $0 < T$; (2) $w_1 + w_2 > T$; (3) $w_1 < T$; (4) $w_1 + w_3 < T$; (5) $w_2 > T$

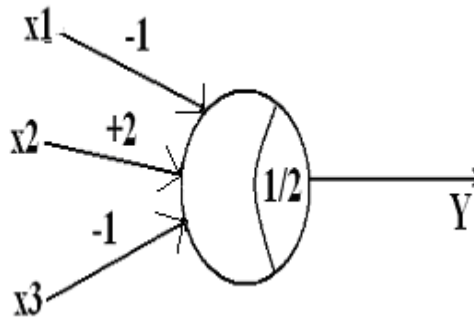
(6) $w_2 + w_3 > T$; (7) $w_3 < T$; (8) $w_1 + w_2 + w_3 < T$

These inequalities are obtained by substituting all eight combinations.

x ₁	x ₂	x ₃
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

The following fig is an example of an LTG realization of the Boolean function

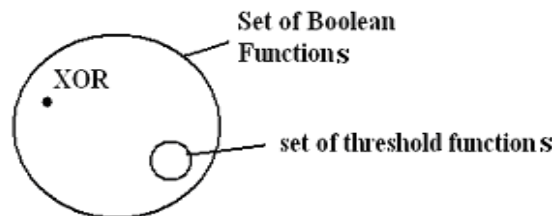
$$Y = \overline{x_1 x_2} + \overline{x_2 x_3}$$



2.3 XOR gate cannot be obtained from signal LTG:-

A Boolean function which can be realized by a single LTG is known as a threshold function. A threshold function is a linearly separable function. Any function that is not linearly separable such as XOR function $y(x_1, x_2) = x_1x_2 + \overline{x_1}\overline{x_2}$ cannot be realized using a signal LTG and is termed a non threshold function.

2.4 Examples of non-threshold functions:



	NUMBER OF THRESHOLD FUNCTIONS B_n	TOTAL NUMBER OF BOOLEAN FUNCTION (2^{2^n})
1.	4	4
2.	14	16
3.	104	256
4.	1,882	65,536
5.	94,572	$4.3 * 10^9$
6.	15,028,134	$1.8 * 10^{19}$
7.	8,378,070,864	$3.4 * 10^{38}$
8.	17,561,539,552,946	$1.16 * 10^{77}$

A single LTG cannot represent all Boolean functions, it is capable of realizing the universal NAND (or NOR) logic operation.

$(x_1 \text{ NAND } x_2 = \overline{x_1 x_2} = \overline{x_1} + \overline{x_2}$ and $x_1 \text{ NOR } x_2 = \overline{x_1 + x_2} = \overline{x_1} \overline{x_2}$). Hence, the LTG is a universal logic gate; any Boolean function is realizable using a network of LTG's (only two logic levels are needed). Besides the basic NAND and NOR function, though an LTG is capable of realizing many more Boolean functions. Therefore a single n-input LTG is a much more powerful gate than a single n-input NAND or NOR gate.

2.5 Advantages of LTG:

1. LT Circuit performs multiplication and division.
2. The number of gates in LT circuits is less than in AON circuits
3. LT circuits are much more powerful than single n-inputs NAND or NOR.
4. Regular Structure.
5. Compact Area. 6. High speed

This can be extended to Quadratic threshold Gates (QTG) and Polynomial Threshold Gates (PTG)

7. Compact Area.
8. High Speed.

2.6 Conclusion: LT Circuits are more Powerful than AON. This can be extended to Quadratic threshold Gates (QTG) and Polynomial Threshold Gates (PTG).

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