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EDGE ODD GRACEFULNESS OF HALIN GRAPHS OF TYPES H (4, n) AND H(5, n)

A. SOLAIRAJU AND A. SATHIK BATCHA

Associate Professor of Mathematics
Jamal Mohamed College, Tiruchirapalli – 620 020.
Tamil Nadu, India. email: solairama@yahoo.co.in

B.T. Assistant in Mathematics,
Khajamian Higher Secondary School
Tiruchirappalli – 620 020, Tamil Nadu, India.

ABSTRACT

A (p, q) connected graph is an edge-odd graceful graph if there exists an injective map f: E(G) → {1, 3, …, $2q$-1} so that induced map f_+: V(G) → {0, 1,2, 3, …, ($2k$-1)}defined by $f_+(x) \equiv \Sigma f(xy)$ (mod $2k$), where the vertex x is incident with other vertex y and $k = \max\{p, q\}$ makes all edges are distinct and odd. In this article, the Edge odd gracefulness of the Halin graph H (4, n) is obtained.
Key words: Graceful Graphs, Edge-odd graceful labeling, Edge-odd Graceful Graph

Introduction: A.Solairaju, A.Sasikala, C.Vimala [2008] got gracefulness of a spanning tree of the graph of product of P_m and C_n, A.Solairaju and K.Chitra [2009] obtained edge-odd graceful labeling of some graphs related to paths. A.Solairaju, C.Vimala, A.Sasikala [2008] gracefulness of a spanning tree of the graph of Cartesian product of S_m and S_n, A. Solairaju et.al. [2010] that the strong product of path P_2 and circuit C_n for all integer n, is an edge-odd graceful.

Section 1 – Preliminaries

The following definitions are needed first for the discussions.

Definition 1.1: Graceful Graph:

A function f of a graph G is called a graceful labeling with m edges, if f is an injection from the vertex set of G to the set {0, 1, 2, …, m} such that when each edge uv is assigned the label $|f(u) - f(v)|$ and the resulting edge labels are distinct. Then the graph G is graceful.

Definition 1.2: Edge-odd graceful graph:

A (p, q) connected graph is an edge-odd graceful graph if there exists an injective map f: E(G) → {1, 3, …, $2q$-1} so that induced map f_+: V(G) → {0, 1, 2, …,($2k$-1)} defined by $f_+(x) \equiv \Sigma f(x, y)$ (mod $2k$), where the vertex x is incident with other vertex y and $k = \max\{p, q\}$ makes all edges are distinct and odd. Hence the graph G is an edge- odd graceful.

Definition 1.3: A Halin graph (4, n) is a connected graph obtained from a cycle C_4 and a cycle C_n and it is defined by the following manner:

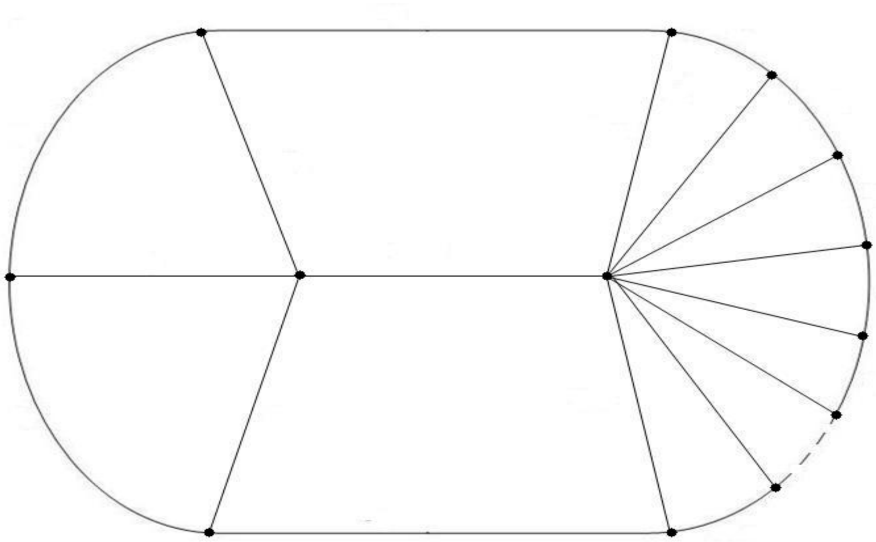
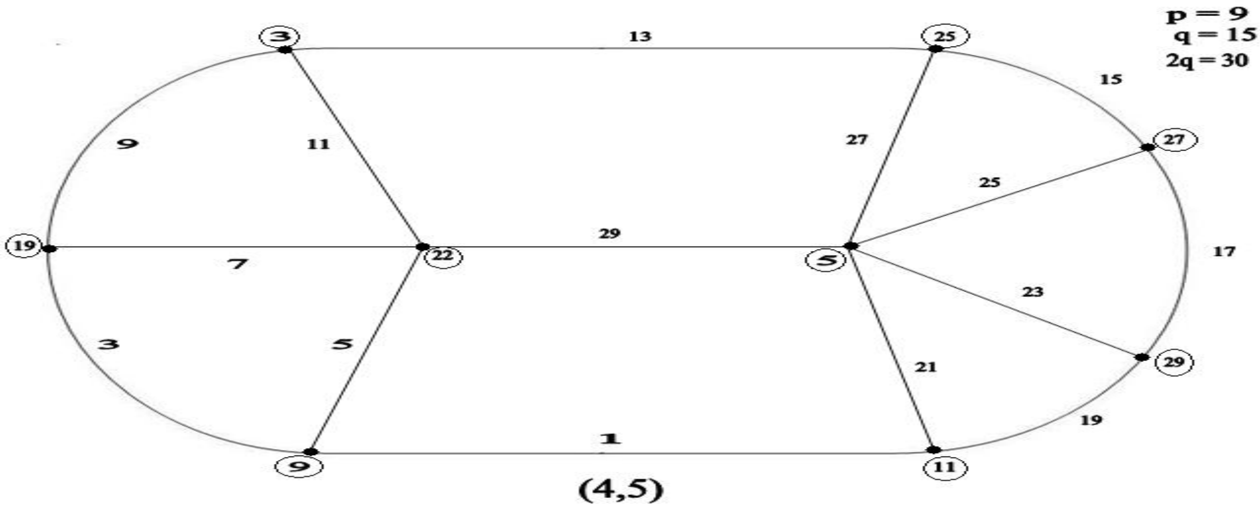


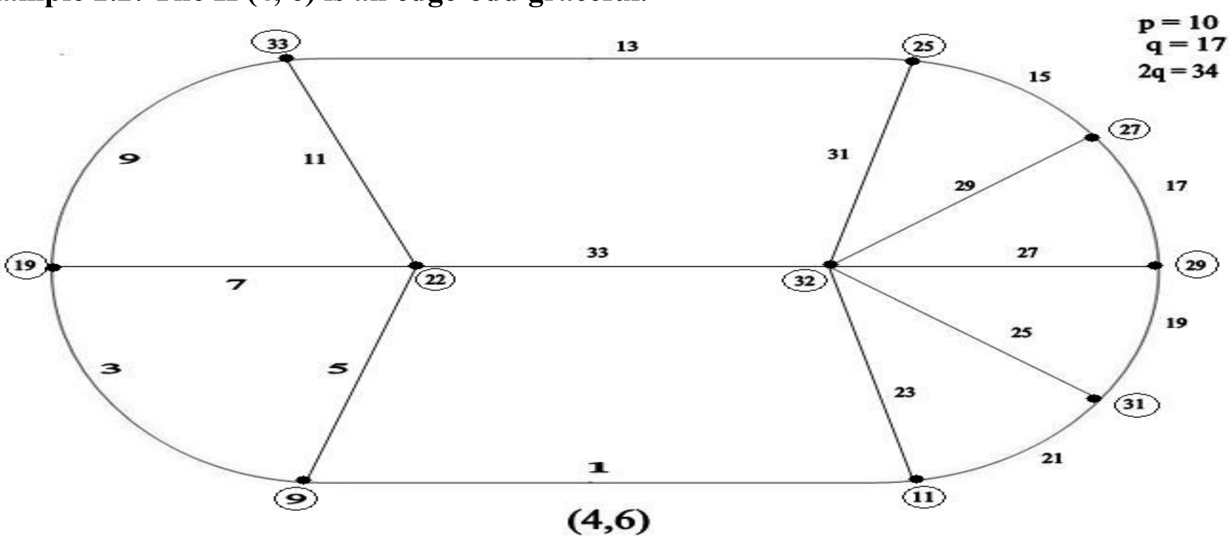
Figure 1: Edge-odd graceful of the Halin graph (4, *n*)

Section 2 - Halin graph H (4, *n*)

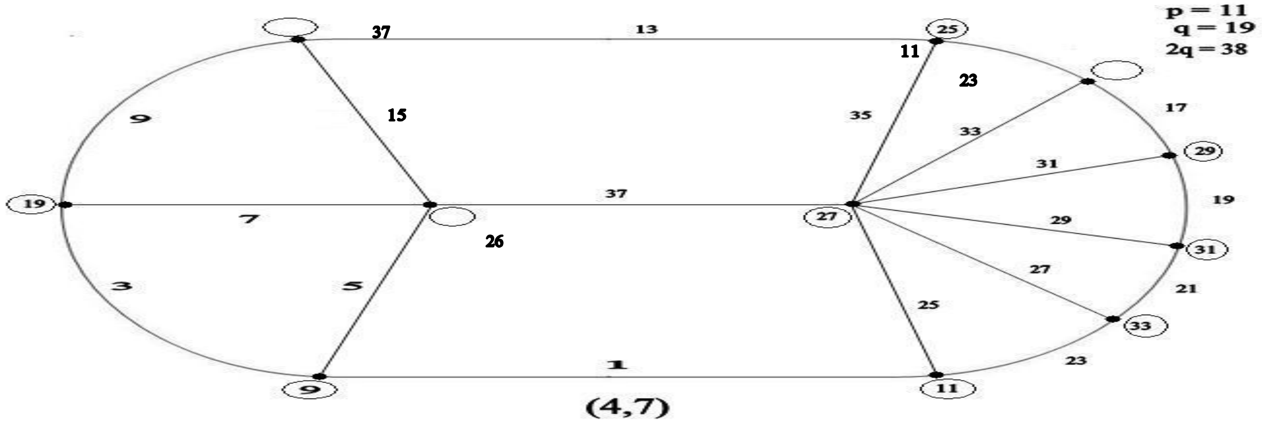
Example 2.1: The H (4, *5*) is an edge-odd graceful.



Example 2.2: The H (4, 6) is an edge-odd graceful.



Example 2.3: The H (4, 7) is an edge-odd graceful.



Theorem 2.4: The Halin graph H (4, *n*) is an edge -odd graceful where *n* ≥ 8.
Proof: The Halin graph (4, *n*) is a connected graph with ($n + 4$) vertices and ($2n + 7$) edges. The arbitrary labeling of edges are required graph is assumed as follows:

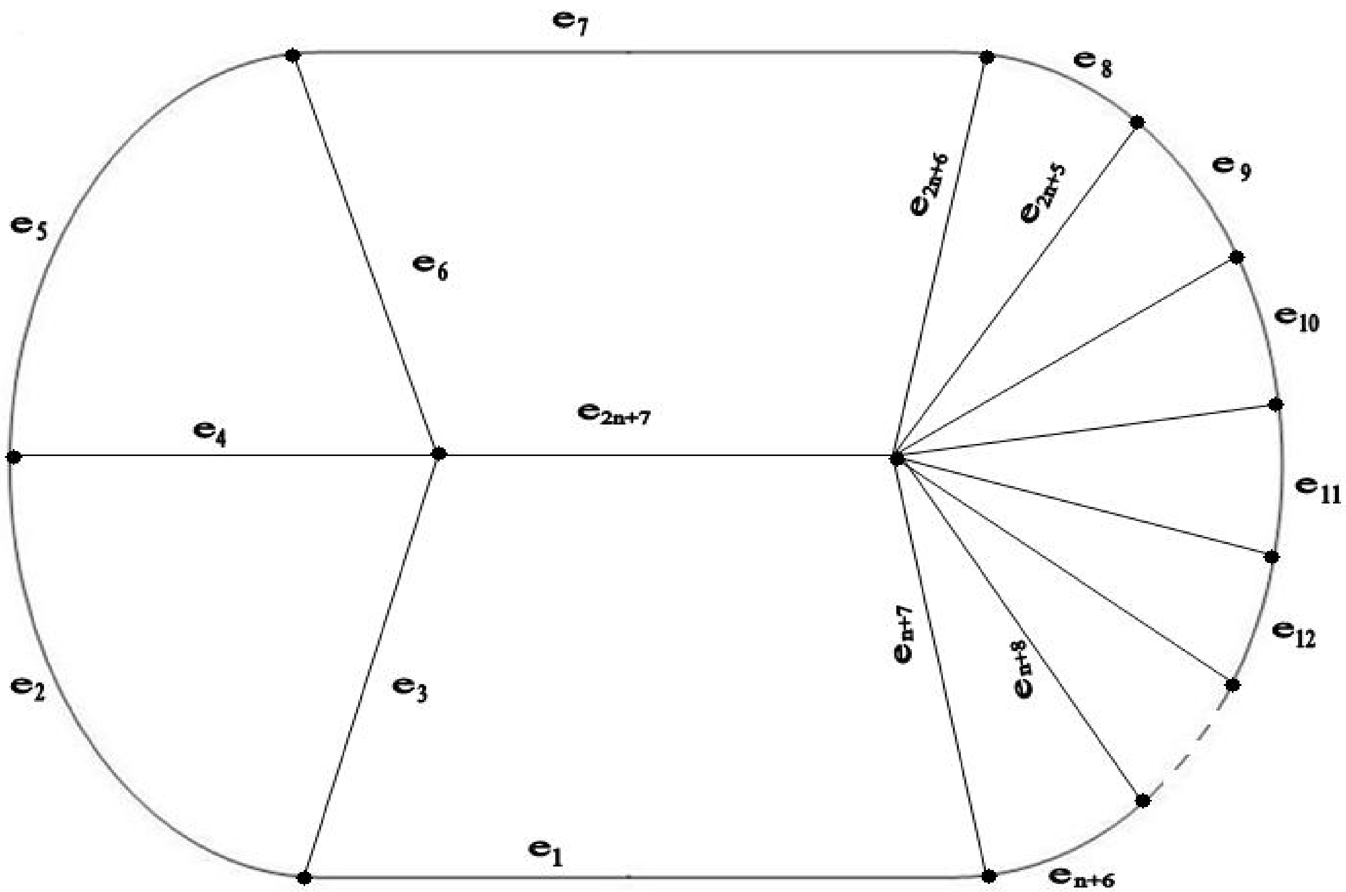
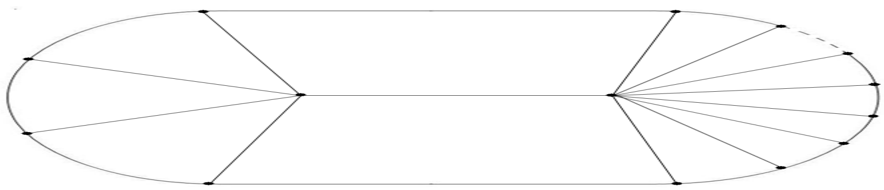


Figure 1: Edge labeling of the Halin graph (4, n).

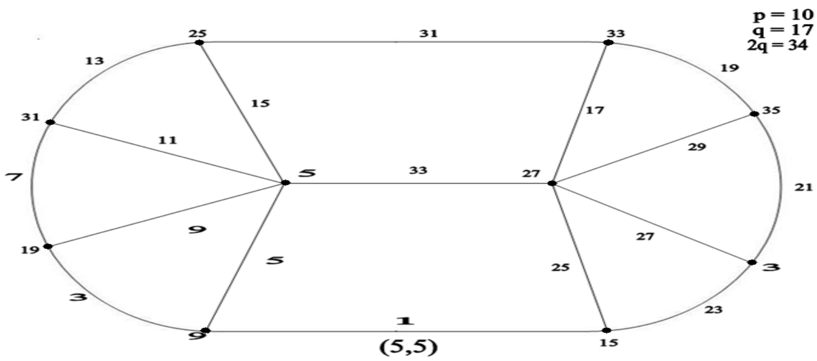
Let k = maximum $\{p, q\}$. Define f: E(G) $\rightarrow$ { 1,2, 3, …, 2k) by $f(e_i) = (2i-1)$ where i varies from 1 to $(2n+7)$; $i \neq 11$, $(2n+3)$; $f(e_{11}) = 4n+5$; $f(e_{2n+3}) = 21$.
Define f_+: V(G) $\rightarrow$ {0, 1, 2, …, (2k-1)} by
$f_+(v) \equiv \Sigma f(uv) \bmod (2k)$, where this sum run over all edges through v ……. Rule (1)

Hence the induced map f_+ provides the distinct labels for vertices and also the edge labeling is distinct. Thus The Halin graph (4, n) is an edge odd - graceful.

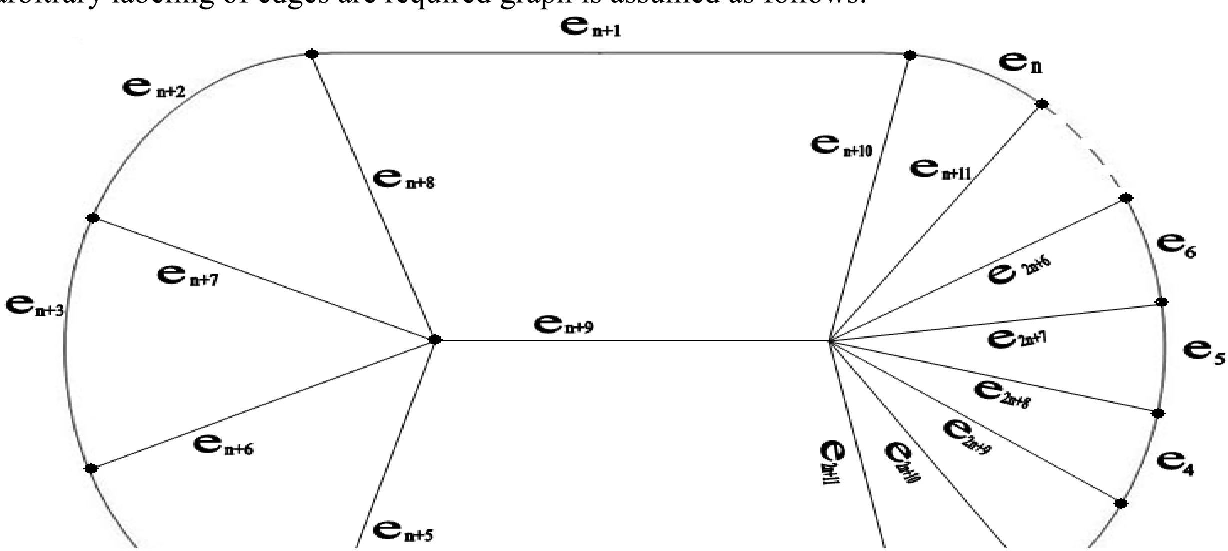
Section 3 - Halin graph H (5, *n*)
The following definition is first started.
Definition 3.1: A Halin graph (5, n) is a connected graph obtained from a cycle C_5 and a cycle C_n and it is defined by the following manner:



Example 3.2: Thus The Halin graph (5, n) is an edge odd - graceful.



Theorem 3.3: The Halin graph H (5, *n*) is an edge -odd graceful where *n* ≥ 5.
Proof: Case (i): *n* ≡ 0, 2, 3 (mod 4)
The Halin graph (5, n) is a connected graph with (n + 5) vertices and (2n + 9) edges.
The arbitrary labeling of edges are required graph is assumed as follows:

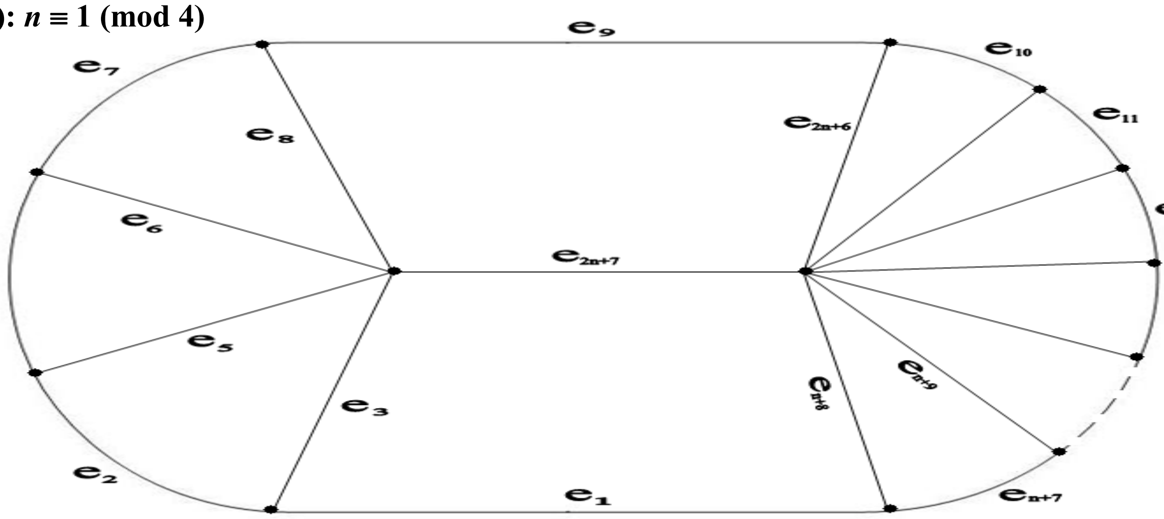


Define f: E(G) → { 1,2, 3, …, 2k) by $f(e_i) = (2i - 1)$ where i varies from 1 to (2n+10); $i \neq (n + 10)$
$f(e_{2n+11}) = 2n + 17$; $f(e_{n+10}) = 4n + 13$.

Define f_+: V(G) → {0, 1, 2, …, (2k-1)} by $f_+(v) \equiv \Sigma f(uv) \bmod (2k)$, where this sum run over all edges through v ……. Rule (1).

Hence the induced map f_+ provides the distinct labels for vertices and also the edge labeling is distinct.

Case (ii): $n \equiv 1 \pmod 4$

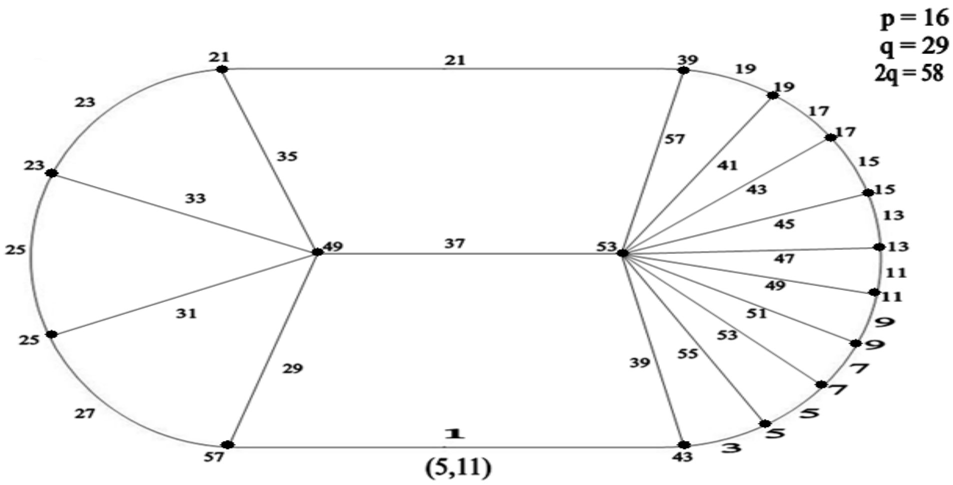


Define f: E(G) → { 1,2, 3, …, 2k) by $f(e_i) = (2i - 1)$ where i varies from 1 to (2n+7); $i \neq (n + 8)$; $i \neq 8$.
$f(e_8) = 2n + 15$; $f(e_{n+8}) = 15$.

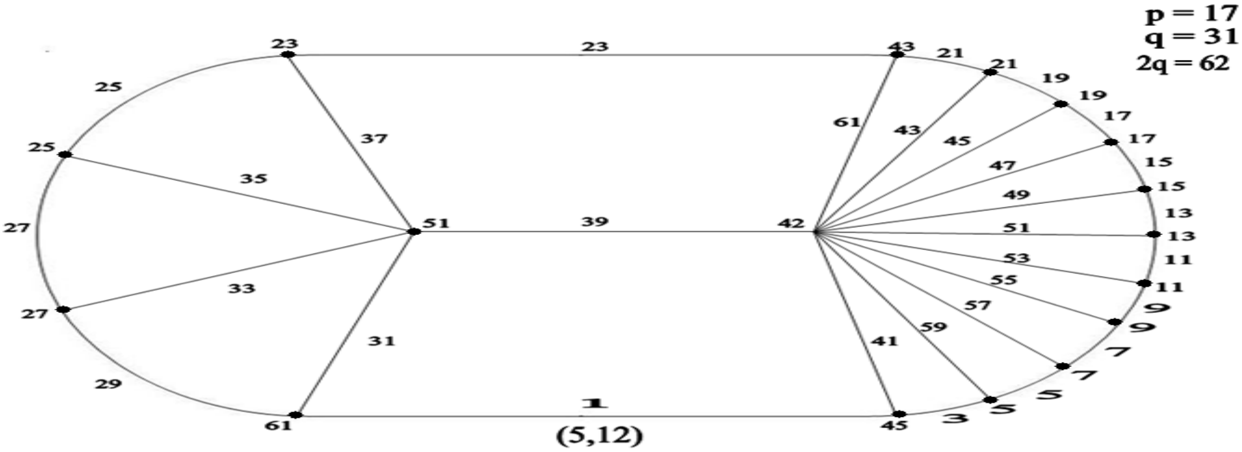
Define f_+: V(G) → {0, 1, 2, …, (2k-1)} by $f_+(v) \equiv \Sigma f(uv) \bmod (2k)$, where this sum run over all edges through v ……. Rule (1).

Hence the induced map f_+ provides the distinct labels for vertices and also the edge labeling is distinct. Thus The Halin graph (5, n) is an edge odd - graceful.

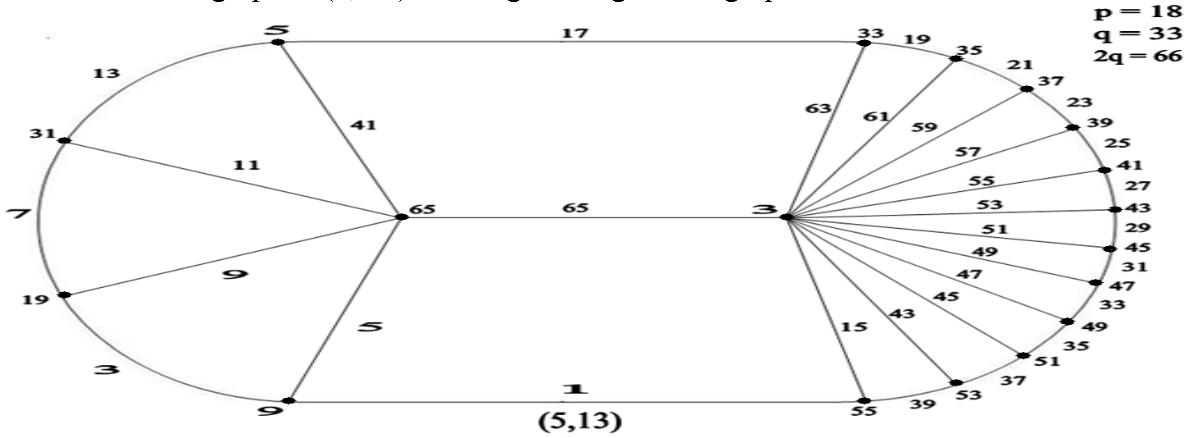
Example 3.4: The Halin graph H (5, 11) and H (5,12) are edge -odd graceful graphs



(5,11)

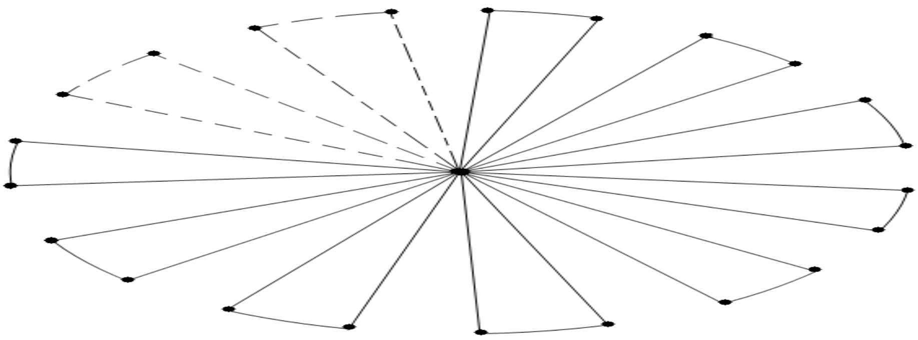


Example 3.5: The Halin graph H (5, 13)is an edge -odd graceful graph.

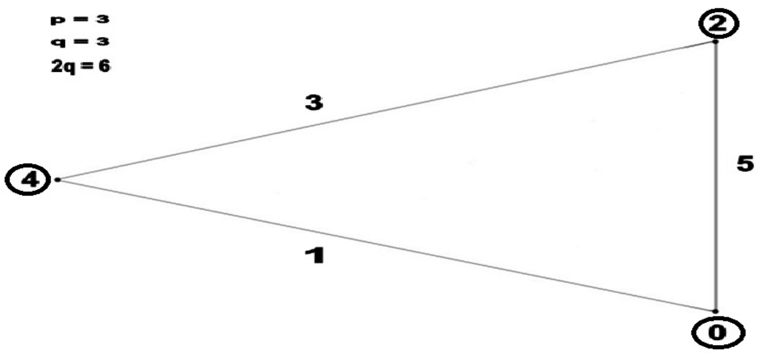


Section 4 - Odd-edge graceful of Friendship graph

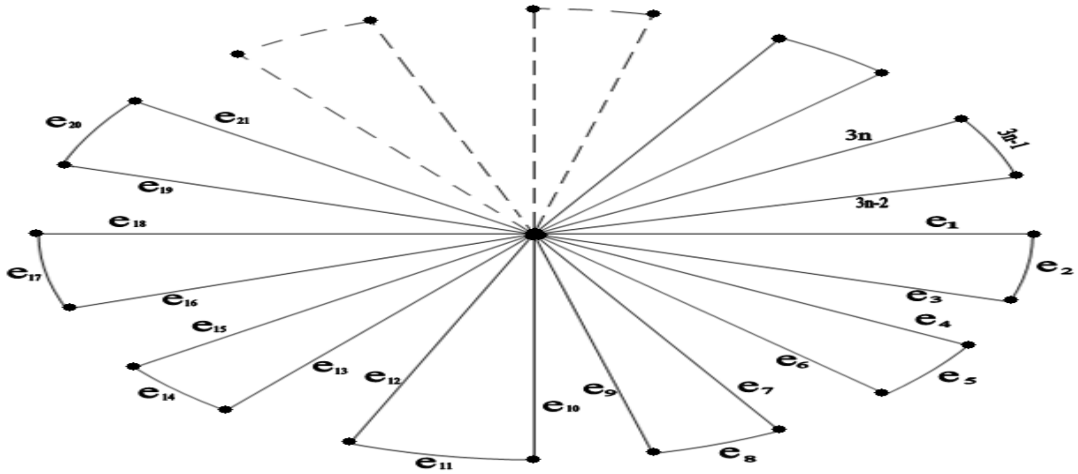
4.1 Definition: The friendship graph F(*n*) is defined by $K_1 + nP_2$.



4.2 Lemma: The friendship graph $K_1 + 1P_2$ is odd-edge graceful.



4.3 Theorem: The friendship graph F(n) = $K_1 + nP_2$ is odd-edge graceful, where n is odd.
Proof: Assume that n is odd. Some arbitrary labeling of all edges in F(n) is as follows:



Define f: E ($K_1 + nP_2$) → { 1, 2, 3, …, 2q} by

$f(e_i) = (2i + 1)$ where i varies from 3, 4, 5, .., (3n-2).

n= 1 (mod 6): f(e_i) = (2i + 1), where i = 1, 2, (3n-1), (3n).

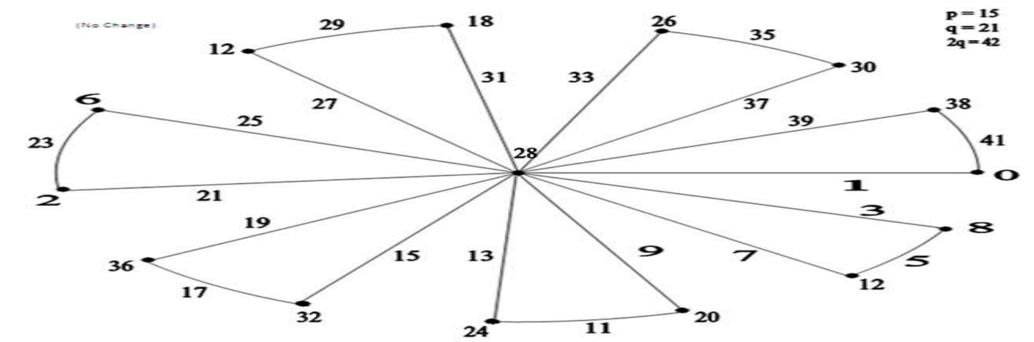
n= 3 (mod 6): $f(e_{3n}) = (6n-1), f(e_{3n-1}) = 1; f(e_1) = 3; f(e_2)= 5$.

n= 5 (mod 6): $f(e_{3n}) = 1, f(e_{3n-1}) = (6n-1); f(e_1) = 5; f(e_2)= 3$.

So f_+: V($K_1 + nP_2$) → {0, 1, 2, …, (2q-1)} by $f_+(v) \equiv \Sigma f(uv) \bmod (2q)$, where this sum run over all edges through v. Hence the induced map f_+ provides the distinct labels for vertices and also the edge labeling is distinct. Thus $K_1 + nP_2$ is an edge odd - graceful.

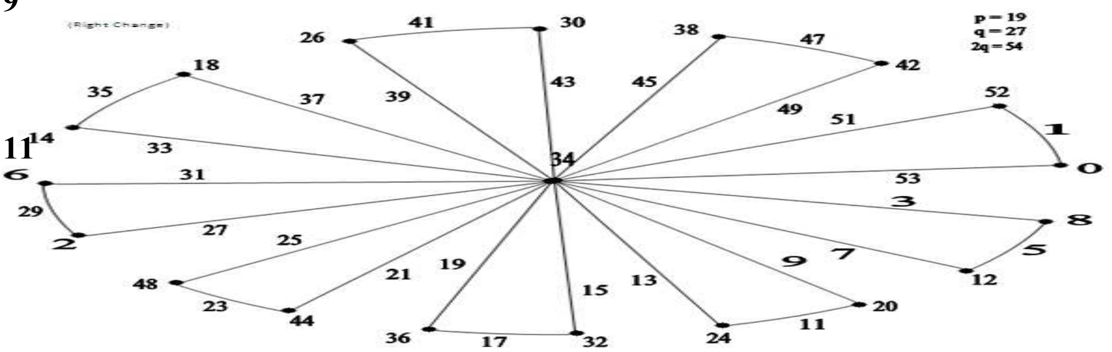
4.4 Example: The friendship graphs $K_1 + 7P_2$, $K_1 + 9P_2$ and $K_1 + 11P_2$ are all odd-edge graceful.

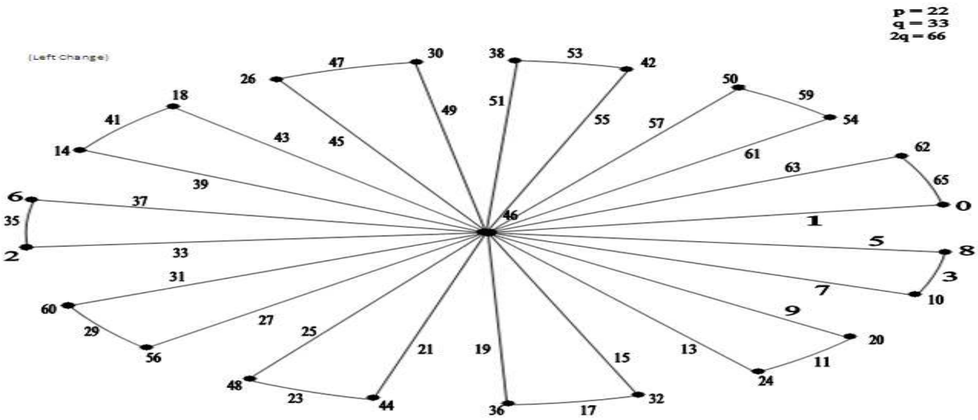
***n* = 7**



***n* = 9**

***n* = 11**





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