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STRUCTURES ON Q-FUZZY GAMMA SUBGROUPS WITH RESPECT TO T-NORMS

A. SOLAIRAJU[1], R. NAGARAJAN[2], & P. MURUGANANTHAM[3]

[1] Associate Professor of Mathematics
Jamal Mohamed College (Autonomous),
Tiruchirappalli – 20.
Tamilnadu, India
Email: solairama@yahoo.co.in

[2] Assistant Professor of Mathematics
J. J. College of Engineering & Technology
Ammapet, Tiruchirappalli – 09.
Tamilnadu, India.
Email: nagalogesh@yahoo.co.in

[3] Head, Department of Mathematics
Kurinji College of Arts & Science, Tiruchirappalli – 02
Tamilnadu, India.
Email: pmananth_kcas@yahoo.in

Abstract: In this Paper, the notion of Q-Fuzzy M-gamma subgroups of groups is introduced and its basic properties are investigated. The study of the homomorphic and Pre-image of Q-Fuzzy M-gamma subgroups are perused. Using t-norm, the notion of sensible Q-Fuzzy M-gamma subgroups in a groups is introduced and some related properties of M-gamma groups are discussed.
Keywords: Q-Fuzzy set, Q - Fuzzy MΓ - group, MΓ - group homomorphism, Imaginable, T-norm, characterized Q-Fuzzy set.

Section 1 – Introduction and Preliminaries

The concept of fuzzy sets was first introduced by Zadeh [1965]. Rosenfeld [1971] used this concept to formulate the notion of Fuzzy groups. Since then, many other fuzzy algebraic concepts based on Rosenfeld's fuzzy groups were developed. Anthony and Sherwood [1969] redefined fuzzy groups in terms of t-norms which is replaced the min operations of Rosenfeld's definition. Using this concept, chang [1968] generalized some of the basic concepts of general topology, Many Researchers [1968] and [1981] applied the concept of fuzzy sets to the elementary theory of Γ - rings. In [1988] Booth introduced the concept of Γ- near rings which is due to satyanarayana [1984]. Also Booth [1990] studied radical theory of a Γ - near ring and introduced the notion of $M\Gamma$ -group. The notion of Intuitionistic Q-Fuzzy semi primality in a semi group is given by Kim[2001]. The notion of Q fuzzy subgroups introduced by [2008], [2010] and [2009]. In this paper, a new class of Q-fuzzy M-gamma subgroup of group are introduced and characterization of some properties of M-gamma groups with respect to t-norms are discussed.

Definition 1.2: A non-empty set 'R' with two binary operations ' + ' and ' • ' is called a near-ring if it satisfies the following axioms.

1. (R, +) is a groups; 2. (R, •) is a semi group;
3. $(a + b) \bullet c = a \bullet c + b \bullet c$, for all $a, b, c \in R$.

Precisely speaking it is a right near-ring. Because it satisfies the right distributive law.
All near rings considered in this paper will be right distributive. A Γ-near ring is a triple (M, +, Γ). where,

(i). (M, +) is a group (not necessarily abelian);

(ii). Γ is a non-empty set of binary operators on M such that, For each $\alpha \in \Gamma$, $(M,+, \alpha)$ is a near ring;

(iii). $a \alpha (b\beta c) = (a \alpha b)\beta c$ for all $a,b,c, \in M$, $\alpha,\beta \in \Gamma$. If, in addition, it holds that

(iv) $a \alpha 0 = 0$ for all $a \in M$; then the Γ-near ring M is said to be zero symmetric.

Definition 1.2: Let G be an additive group. If, for all $a,b \in M$, $\alpha,\beta \in \Gamma$ and $x \in G$ it holds that

(i) $a \alpha x \in G$

(ii) $a \alpha(b\beta c) = (a \alpha b)\beta x$

(iii) $(a+b) \alpha x = a \alpha x + b \alpha x$, then G is called an M-Gamma group or MΓ -group.

In what follows, let M denotes the Γ -near ring and G denotes the MΓ -group unless or otherwise specified.

Definition 1.3: A subgroup H of G for which $a \alpha h \in K$ for $a \in M$, $\alpha \in \Gamma$, $h\in H$ is called an MΓ-subgroup of G.

We now review some fuzzy logic concepts. A fuzzy set in a set G is a function $A : G \to [0, 1]$. We shall use the notation $U(A;t)$, called a level subset of A for which $\{x \in G / A(x) \geq t\}$ where $t \in [0,1]$.

Definition 1.4: Let Q and G a set and a group respectively. A mapping $A : G \times Q \to [0, 1]$ is called Q-Fuzzy set in G.

Definition 1.5 :A Q-fuzzy set 'A' in G is called a Q-fuzzy MΓ-subgroup of G if,

(i) $A(x-y, q) \geq \text{Min} \{ A(x,q), A(y,q)\}$

(ii) $A(a\, \alpha\, x,q) \geq A((x,q)$, for $a \in M$, $q \in Q$, $x \in G$ and $\alpha \in \Gamma$.

Definition 1.6: By a t-norm T, we mean a function $T : [0,1] \times [0,1] \to [0,1]$ satisfying the following conditions

$[T_1]$ $T(x, 1) = x$

$[T_2]$ $T(x, y) \leq T(x, z)$ if $y \leq z$

$[T_3]$ $T(x, y) = T(y, x)$

$[T_4]$ $T(x, T(y, z)) = T(T(x, y), z)$ for all $x, y, z \in [0, 1]$.

Proposition 1.7: For a t-norm, the following statement holds $T(x,y) \leq \text{Min} \{x,y\}$, for all $x,y \in [0,1]$. For a t-norm T on [0,1], denoted by ΔT the set of elements $\alpha \in [0,1]$ such that $T(\alpha, \alpha) = \alpha$, (ie) $\Delta T = \{ \alpha \in [0,1] / T(\alpha, \alpha) = \alpha \}$

Definition 1.8: Let T be a t-norm. A fuzzy set A in G is said to satisfy idempotent property with respect to T, if $\text{Im}(A) \subseteq \Delta T$.

In Definition (2.5) we use T operator for min operation.

Section 2 - Properties of Q-Fuzzy MΓ-subgroups of Group

In this section, The notion of Q-fuzzy MΓ-subgroups of MΓ-group are discussed.

Proposition 2.1: Let T be a t-norm of 'A' is idempotent Q-fuzzy MΓ-subgroup of G. Then $A(0, q) \geq A(x, q)$ for all $x \in G$, $q \in Q$.

Proof: For every $x \in G$ and $q \in Q$, it follows that

$A(0,q) = A(x-x, q) \geq T\{A(x,q), A(x,q)\} = A(x,q)$.

Proposition 2.2: If 'A' is an idempotent Q-fuzzy M-gamma subgroup of G, then the set $G^p = \{x \in G \,/\, A(x,q) \geq A(p,q)\}$ is an M-gamma subgroup of G.

Proof: Let $x,y \in G^p$, then $A(x,q) \geq A(p,q)$ and $A(y,q) \geq A(p,q)$. since 'A' is an Q-fuzzy MΓ-subgroup of G, it follows that $A(x-y,q) \geq \text{Min}\ \{(x,q), A(y,q)\} \geq \text{Min}\ \{A(p,q), A(p,q)\} = A(p,q)$. Now let $a \in M$, $\alpha \in \Gamma$ and $h \in G^p$ then $A(x\alpha\ h,q) \geq A(h,q) \geq A(p,q)$ then we have $A(x-y,q) \geq A(p,q)$ and $A(a\alpha x,q) \geq A(p,q)$ that $x-y \in G^p$ and $a\alpha h \in G^p$. This completes the proof.

Corollary 2.3: Let T be a t-norm. If 'A' is an idempotent Q-fuzzy MΓ-subgroup of G, then the set $A_G = \{x \in G \,/\, A(x,q) = A(0,q)\}$ is an MΓ-subgroup of G.

Proof: From the proposition (2.1), $A_G = \{x \in G \,/\, A(x,q) = A(0,q)\} = \{x \in G \,/\, A(x,q) \geq A(p,q)\}$. Hence A_G is an MΓ-subgroup of G from the Proposition (3.2).

Definition 2.4: Let G and G' be MΓ-groups. A map θ: G → G' is called a MΓ-group homomorphism if $\theta\ (x+y) = \theta\ (x) + \theta\ (y)$ and $\theta(a\ \alpha\ x) = a\ \alpha\ \theta\ (x)$ for all $a \in M$, $\alpha \in T$ and $x \in G$.

Definition 2.5: Let θ : G → G' be an MΓ-group homomorphism's of MΓ-groups. For a Fuzzy set A in G', we define a characterized Q-fuzzy set A^θ in G by $A^\theta\ (x,q) = A\ (\theta\ (x,q))$ for all $x \in G$.

Propositions 2.6: Let θ : G → G' be an MΓ-group homomorphism's of MΓ-groups. If 'A' is an Q-fuzzy MΓ-subgroup of G', then A^θ is an Q-fuzzy MΓ-subgroup of G.

Proof: For any $x,y \in G$ and $q \in Q$, we have

$$\begin{aligned} A^\theta(x-y,q) &= A\ (\theta\ (x-y,q)) \\ &= A\ ((\theta\ (x) - \theta\ (y),q)) \\ &\geq T\{A\ (\theta\ (x,q), A\ (\theta\ (y,q))\} \\ &\geq T\{A^\theta\ (x,q), A^\theta\ (y,q)\} \end{aligned}$$

Let $a \in M$, $\alpha \in T$ and $x \in G$, then

$$\begin{aligned} A^\theta\ (a\ \alpha\ x,q) &= A\ (\theta\ (a\ \alpha\ x,q)) \\ &= A\ (a\ \alpha\ \theta\ (x,q)) \\ &\geq A\ (\theta\ (x,q) \end{aligned}$$

$\geq A^{\theta}(x,q)$

This completes the proof.

Proposition 2.7: Let I be an MΓ-subgroup of G and let 'A' be a Q-fuzzy set in G defined by

A(x,q) = (a,q) if x ∈ I

(b,q) otherwise.

For all x ∈ G, and a, b ∈ [0, 1] with a > b, then 'A' is Q-fuzzy MΓ-subgroup of G where min{a, b} = max {a + b - 1, 0} for all a, b ∈ [0, 1].

Proof: Let x , y ∈ G. If x, y ∈ I. Then

min {A(x,q), A(y,q)} = min {a,a} = max {2a-1,0} = 2a, if a ≥ ½ ; b, if a < ½

≤ a = A(x-y, q).

For all p ∈ M and a ∈ Γ, it follows that

A(pax, q) = A(x,q) = a. If y ∈ I and x ∉ I (or x ∈ I and y ∉ I), then

min {A(x,q), A(y,q)} = min {a,b} = max {a + b - 1, 0}; = a + b - 1 if a + b ≥ ½; b, otherwise.

≤ b = A(x-y, q).

For all p ∈ M and α ∈ T, A(pax, q) ≥ b = A(y,q).

If y ∉ I and x ∉ I, then min {A(x,q), A(y,q)} = A(b,b) = max {2b-1, 0} ; = 2b-1, if b ≥ ½ ; 0,. otherwise.

For all p ∈ M and α ∈ T; it gives that A(pαx, q) ≥ b = A(x,q). Hence 'A' is an Q-fuzzy set subgroup of G. For a subset I of MΓ-subgroup of G. Φ denotes the characteristic function of I.

Corollary 2.8: I ⊆ G, then I is an MΓ- subgroup of G if and only if Φ is an Q-fuzzy MΓ-subgroup of G.

Proof: Let I be an MΓ-subgroup of G, then it is easy to show that Φ is an Q-fuzzy MΓ-subgroup of G. In fact, let x,y ∈ I and so x-y ∈ I.

Hence we have Φ(x-y,q) = 1 = T{ Φ(x,q), Φ(y,q)} = T{1,1}. Assume that x ∈ I. and y∈ I. or x ∉ I and y∈ I., then Φ(x,q) = 1 >0 = Φ(y,q) (or Φ(x,q)=0<1= Φ(y,q)).

It follows that Φ(x-y,q)≥0 T{ Φ(x,q), Φ(y,q)} = min{1,0} = 0. Now let a∈ M and α ∈ Γ. If y∈ I., then we have a α x ∈ I. Hence Φ(a α x,q) = 1 = Φ(y,q).

If $y \notin I$, then $\Phi(a\ \alpha\ x,q) \geq \Phi(y,q)$. Conversly let Φ be an Q-fuzzy MΓ-subgroup of G. Let $x,y \in I$. Then we have $\Phi(x-y,q) \geq T\{\Phi(x,q), \Phi(y,q)\} = T\{1,1\} = 1$ and so $x-y \in I$. . Now let $a \in M$ and $\alpha \in \Gamma$ and $y \in I$. Hence $\Phi(a\ \alpha\ x,q) \geq \Phi(y,q) = 1$ and so $a\ \alpha\ x \in I$.

Proposition 2.9: Let T- be a t-norm then every idempotent Q-fuzzy MΓ-subgroup of G is a fuzzy ideal of G.

Proof: Let 'A' be an idempotent Q-fuzzy MΓ-subgroup of G then $A(x-y,q) \geq T\{A(x,q), A(y,q)\}$ for all $x,y \in G$. Since 'A' satisfies the idempotent property. it gives that

$$\begin{aligned} \text{Min}\{A(x,q), A(y,q)\} &= T\{\text{Min}\{A(x,q), A(y,q)\}, \text{Min}\{A(x,q), A(y,q)\}\} \\ &\leq T\{A(x,q), A(y,q)\} \\ &\leq \text{Min}\{A(x,q), A(y,q)\}, \end{aligned}$$

Thus $A(x-y,q) \geq T\{A(x,q), A(y,q)\} = \text{Min}\{A(x,q), A(y,q)\}$ so that 'A' is a fuzzy ideal of G.

Proposition 3.10: The family of Q-fuzzy MΓ-subgroups of G is a completely distributive lattice with respect to meet '$\wedge$' and join '$\vee$'.

Proof: Since [0,1] is a completely distributive lattice with respect to the usual ordering in [0,1], it is sufficient to show that $\vee_{\alpha \in \wedge} A_\alpha$ and $\wedge_{\alpha \in \wedge} A_\alpha$ are Q-fuzzy MΓ-subgroups of G for a family of Q-fuzzy MΓ-subgroups $\{A_\alpha / \alpha \in \wedge\}$.

For any $x,y \in G$, we have

$$\begin{aligned} (\vee_{\alpha \in \wedge} A_\alpha)(x-y,q) &= \text{Sup}\,(A_\alpha(x-y,q) / \alpha \in \wedge\} \\ &\geq \text{Sup}\,\{T(A_\alpha(x,q), A_\alpha(y,q) / \alpha \in \wedge \\ &\geq T\{\text{Sup}\,\{T(A_\alpha(x,q) / \alpha \in \wedge, \text{Sup}\{A_\alpha(y,q) / \alpha \in \wedge \\ &= T\{(\vee_{\alpha \in \wedge} A_\alpha)(x,q), (\vee_{\alpha \in \wedge} A_\alpha)(y,q)\} \end{aligned}$$

$$\begin{aligned} (\wedge_{\alpha \in \wedge} A_\alpha)(x-y,q) &= \inf\,(A_\alpha(x-y,q) / \alpha \in \wedge \\ &\geq \inf\,\{T(A_\alpha(x,q), A_\alpha(y,q) / \alpha \in \wedge\} \end{aligned}$$

$\geq T\{\inf\{T(A_\alpha(x,q) / \alpha \in \Lambda, \inf\{A_\alpha(y,q) / \alpha \in \Lambda\}$

$= T\{(\wedge_{\alpha\in\Lambda} A_\alpha)(x,q), (\wedge_{\alpha\in\Lambda} A_\alpha)(y,q)\}$

Now let $a \in M$, $y \in I$ and $\alpha \in \Gamma$, then

$(\vee_{\alpha\in\Lambda} A_\alpha)(a\,\alpha\,x,q) = \text{Sup}\,(A_\alpha(a\,\alpha\,x,q) / \alpha \in \Lambda)$

$\geq \text{Sup}\,(A_\alpha(x,q) / \alpha \in \Lambda)$

$= (\vee_{\alpha\in\Lambda} A_\alpha)(x,q)$

$(\wedge_{\alpha\in\Lambda} A_\alpha)(a\,\alpha\,x,q) = \inf(A_\alpha(a\,\alpha\,x,q) / \alpha \in \Lambda)$

$\geq \inf(A_\alpha(x,q) / \alpha \in \Lambda)$

$= (\wedge_{\alpha\in\Lambda} A_\alpha)(x,q)$

Hence $\vee_{\alpha\in\Lambda} A_\alpha$ and $\wedge_{\alpha\in\Lambda} A_\alpha$ are Q-fuzzy MΓ-subgroups of G. This completes the proof.

Proposition 3.11: Let 'A' be an Q-fuzzy MΓ-subgroups of G and let $\alpha \in \Gamma$ be such that $T(\alpha,\alpha) = \alpha$. Then $\cup(A;\alpha)$ is either empty or an MΓ-subgroups of G for all $x \in G$.

Proof: Let $x, y \in \cup(A;\alpha)$. Then $A(x,q) \geq \alpha$ and $A(y,q) \geq \alpha$

and so $A(x-y,q) \geq T(A(x,q), A(y,q)) \geq T(\alpha,\alpha) = \alpha$ which implies that $x-y \in \cup(A;\alpha)$.

Now let $a \in M$, $y \in \cup(A;\alpha)$ and $\gamma \in \Gamma$. Then $A(a\,\gamma\,x,q) \geq A(x,q) \geq \alpha$. so $a\,\gamma\,x \in \cup(A;\alpha)$.

Hence $\cup(A;\alpha)$ is MΓ-subgroups of G, since $T(1,1) = 1$ we have the following corollary.

Corollary 2.12 : If 'A' is an Q-fuzzy MΓ-subgroups of G then $\cup(A:1)$ is either empty or an MΓ-subgroups of G.

Proof: For a family $\{A_\alpha / \alpha \in \Lambda\}$ Q-fuzzy sets in G, define the join $\vee_{\alpha\in\Lambda} A_\alpha$ and the meet $\wedge_{\alpha\in\Lambda} A_\alpha$ as follows

$(\vee_{\alpha\in\Lambda} A_\alpha)(x,q) = \text{Sup}\,(A_\alpha(x,q) / \alpha \in \Lambda)$

$(\wedge_{\alpha\in\Lambda} A_\alpha)(x,q) = \inf(A_\alpha(x,q) / \alpha \in \Lambda)$

for all $x \in G$, where Λ is any index set.

Proposition 3.13.: Let T be a t-norm and let 'A' be a Q-fuzzy set in G with Im (A) = $\{\alpha_1, \alpha_2, \ldots \ldots \ldots \alpha_n\}$ where $\alpha_i < \alpha_j$ where $i > j$. Suppose that there exists a chain of MΓ-subgroups of G: $G_0 \subset G_1 \subset \ldots \ldots \ldots \ldots \subset G_n = G$ such that $A(\bar{G}_k) = \alpha_k$; where $\bar{G}_k = G_k / G_{k-1}$ and $G_1 = 0$ for $k = 0, 1, 2, \ldots, n$. Then 'A' is an Q-fuzzy MΓ-subgroups of G.

Proof: Let $x, y \in G$. If x and y belong to the some $\bar{G}_k$. Then $A(x,q) = A(y,q) = \alpha_k$ and $x\text{-}y \in G_k$.

Hence $A(x\text{-}y,q) \geq \alpha_k = \text{Min}\,\{A(x,q), A(y,q)\}$

$\geq T\{A(x,q), A(y,q)\}$

Let $x \in \bar{G}_i$ and $y \in \bar{G}_j$ for every $i \neq j$. Without loss of generality, assume that $i > j$;

Then $A(x, q) = \alpha_i < \alpha_j = A(y, q)$ and $x - y \in G_i$.

It follows that

$A(x\text{-}y,q) \geq \alpha_i = \text{Min}\,\{A(x,q), A(y,q)\}$

$\geq T\{A(x,q), A(y,q)\}$

Now let $y \in G$, then there exists G_k such that $y \in \bar{G}_k$ for some $k \in \{0,1,2,\ldots\}$. For any $a \in M$, $x \in \bar{G}_k$ and $\alpha \in \Gamma$. We have $a\,\alpha\,x \in G_k$ and so

$A(a\,\alpha\,x,q) \geq \alpha_k \geq A(x,q)$. Hence 'A' is an Q-fuzzy MΓ-subgroups of G.

Proposition 2.14.: Let T be a t-norm then every Imaginable Q-fuzzy MΓ-subgroups of G is a fuzzy MΓ-subgroup of G.

Proof: Assume 'A' is imaginable Q-fuzzy MΓ-subgroups of G. Then $A(x\text{-}y,q) \geq T\{A(x,q), A(y,q)\}$ and $A(a\,\alpha\,x,q) \geq A(x,q)$ for all x,y in G.

Since A is imaginable, it follows that

$\text{Min}\{A(x,q), A(y,q)\} = T\{\text{Min}\{A(x,q), A(y,q), \text{Min}\{A(x,q), A(y,q)\}\}$

$\leq T\{A(x,q), A(y,q)\}$

$\leq \text{Min}\,(A(x,q), A(y,q)\}$ and so

$T(A(x,q), A(y,q)) = \text{Min}\{A(x,q), A(y,q)\}$.

It follows that $A(x-y,q) \geq T\{A(x,q), A(y,q)\} = Min\{A(x,q), A(y,q)\}$ for all $x,y \in G$. Hence 'A' is fuzzy MΓ-subgroups of G.

Proposition 2.15: If 'A' is a Q-fuzzy MΓ-subgroups of a MΓ-group G and θ is an endomorphism of G, then $A_{[\theta]}$ is a Q-fuzzy MΓ-subgroups of G.

Proof: For any $x, y \in G$, it gives that

1. $A_{[\theta]}(x-y,q) = A[\theta(x,q), \theta(y,q)]$
$\geq T\{A[\theta(x,q)], A[\theta(y,q)]\}$
$= T\{A_{[\theta]}(x,q), A_{[\theta]}(y,q)\}$
2. $A_{[\theta]}(a\,\alpha\,x,q) = A[\theta(a\,\alpha\,x,q)]$
$\geq A[\theta(x,q)]$
$= A_{[\theta]}(x,q)$

Hence $A_{[\theta]}$ is a Q-fuzzy MΓ-subgroups of G.

Definition 2.16: Let $f : G \to G'$ be a group homomorphism and 'A' be a Q-fuzzy MΓ-subgroups of G' then $A\,f(x,q) = (A_0 f)(x,q) = f^{-1}(A(x,q))$

Proposition 2.17: Let $f : G \to G'$ be a group homomorphism and 'A' be a Q-fuzzy MΓ-subgroups of G' then $f^{-1}(A)$ is Q-fuzzy MΓ-subgroups of G.

Proof: Let $x, y \in G$. It follows that

$f^{-1}(A)(x-y,q) = (A°f)(x-y, q)$
$= Af(x-y,q)$
$= A((f(x)-f(y), q)$
$\geq T\{A(f(x, q), A(f(y, q)\}$
$\geq T\{(A°f)(x, q), (A°f)(y, q)\}$
$\geq T\{f^{-1}(A)(x, q), f^{-1}(A)(y, q)\}$

$f^{-1}(A)(a\,\alpha\,x, q) = (A°f)(a\,\alpha\,x, q)$
$= Af(a\,\alpha\,x, q)$
$\geq Af(x, q)$
$\geq (A°f)(x, q)$
$\geq f^{-1}(A)(x, q).$

Proposition 3.18: Let 'A' be Q-fuzzy MΓ-subgroups of G and A* be a Q-fuzzy set in G defined by A*(x, q) = A(x, q) + 1 – A(e, q). Then A* is Q-fuzzy MΓ-subgroups of G containing A.

Proof: For x, y ∈ G, it gives that

$$\begin{aligned} A^*(x-y,q) &= A(x-y,q) + 1 - A(e,q) \\ &\geq T\{A(x,q), A(y,q)\}+1 - A(e,q) \\ &\geq T\{A(x,q) +1 - A(e,q),\ A(y,q)+1 - A(e,q)\} \\ &\geq T\{A^*(x,q), A^*(y,q)\} \end{aligned}$$

$$\begin{aligned} A^*(a\ \alpha\ x,q) &= A(a\ \alpha\ x,q) + 1 - A(e,q) \\ &\geq \{A(x,q)+1 - A(e,q) \\ &\geq \{A^*(x,q)\}. \end{aligned}$$

Proposition 2.19: Let T be a continuous t-norm and let 'f' be a homomorphism on a group G. If 'A' is Q-fuzzy MΓ-subgroups of G, then A^f is Q-fuzzy MΓ-subgroup of f(G).

Proof: Let $A_1 = f^{-1}(y_1,q)$, $A_2 = f^{-1}(y_2,q)$ and $A_{12} = f^{-1}(y_{12},q)$, where $y_1, y_2 \in f(R)$, $x \in Q$.

Consider the set

$A_1 - A_2 = \{x \in S / (x,q) = (a_1,q) - (a_2,q)\}$ for some $(a_1,q) \in A_1$ and $(a_2,q) \in A_2$

If $(x,q) \in A_1 - A_2$, then $(x,q) = (x_1,q) = (x_2,q)$ for some $(x_1,q) \in A_1$ and $(x_2,q) \in A_2$ so that we have

$f(x,q) = f(x_1,q) - f(x_2,q) = y_1 - y_2$

(ie) $(x,q) \in f^{-1}(y_1,q) - f(y_2,q) = f^{-1}(y_1-y_2,q) = A_{12}$

Thus $A_1 - A_2 \subset A_{12}$, it follows that

$$\begin{aligned} \text{(i) } A^f(y_1-y_2, q) &= \text{Sup } \{A(x, q) / (x, q) \in f^{-1}[(y_1, q) - (y_2, q)]\} \\ &= \text{Sup } \{A(x,q) / (x,q) \in A_{12}\} \\ &\geq \text{Sup } \{A(x,q) / (x,q) \in A_1 - A_2\} \\ &\geq \text{Sup } \{A(x_1,q) - (x_2,q) / (x_1,q) \in A_1 \text{ and } (x_2,q) \in A_2\} \\ &\geq \text{Sup } \{T\{A(x_1,q), A(x_2,q)\} / (x_1,q) \in A_1 \text{ and } (x_2,q) \in A_2\}. \end{aligned}$$

Since T is continuous, for every $\in > 0$, and

$\text{Sup}\{\{A(x_1,q) / (x_1,q) \in A_1\} - (x_1^*,q)\} \leq \delta$ and $\text{Sup}\{\{A(x_2,q) / (x_2,q) \in A_2\} - (x_2^*,q)\} \leq \delta$, it gives that $T\{\text{Sup}\{\{A(x_1,q) / (x_1,q) \in A_1\}, \text{Sup}\{\{A(x_2,q) / (x_2,q) \in A_2\} - T((x_1^*,q), (x_2^*,q)) \leq \varepsilon$

Choose $(a_1,q) \in A_1$ and $(a_2,q) \in A_2$ such that $\text{Sup}\{\{A(x_1,q) / (x_1,q) \in A_1\} - A(a_1,q)\} \leq \delta$ and

$\text{Sup}\{\{A(x_2,q) / (x_2,q) \in A_2\} - A(a_2,q)\} \leq \delta$.

Then $T\{\text{Sup}\{\{A(x_1,q) / (x_1,q) \in A_1\}, \text{Sup}\{\{A(x_2,q) / (x_2,q) \in A_2\} \; T((a_1,q),(a_2,q)) \leq \varepsilon$.

Consequently, $A^f(y_1-y_2,q) \geq \text{Sup}\{T\{A(x_1,q), A(x_2,q)\} / (x_1,q)\in A_1 \text{ and } (x_2,q)\in A_2\}$

$\geq T\{\text{Sup}\{A(x_1,q) / (x_1,q)\in A_1\}, \{A(x_2,q) / (x_2,q)\in A_2\}$

$\geq T\{A^f(y_1,q), A^f(y_2,q)\}$

Similarly we can show $A^f(a\,\alpha\,x,q) \geq A^f(x,q)$. Hence A^f is Q-fuzzy MΓ-subgroup of G.

Conclusion: Group theory has vast and potential applications in many core areas like physics, chemistry, communications, coding theory, computer science etc. [11] Introduced the concept of Q fuzzy group. In this paper we investigated the concept of Q-fuzzy MΓ-subgroups with respect to t-norms and characterization of them.

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