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COMPACTNESS IN SMALLEST NEIGHBORHOOD SPACES

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Abstract

In this paper, join operator of two topological spaces is discussed.Properties of extended Khalimsky line is also studied.

Keywords.

Alexandrov space, extended Khalimsky line, join operator, smallest neighborhood spaces.

1 Introduction

In any topological space, the intersection of a finite family of open sets is open, whereas the stronger requirement that an arbitrary intersection of open sets be open is not satisfied in general. Alexandrov (1937) considers topological spaces where arbitrary intersection of open sets are open. Such spaces are called smallest neighborhood spaces or Alexandrov spaces. A topological space stored in a computer is finite. Spaces where points can have an infinite number of neighbors seems less likely to appear in computer applications.

1.1 Definition (4) : A smallest neighborhood space is called locally finite if every point in it has a finite adjacency neighborhood. If every point has countable adjacency neighborhood it is called locally countable.

2 The Join Operator.

By using join operator we can combine two topological spaces.

2.1 Definition (4)

Let X and Y be two topological spaces. The join of X and Y denoted $X \vee Y$ is a topological space over the disjoint set union of X and Y where a subset $A \subset X \cup Y$ is open if either

1 $A\cap X$ is open in X and $A \cap Y = \phi$ or

2 $A\cap X = X$ and $A\cap Y$ is open in Y

A set $B \subset X\vee Y$ is closed if and only if

1 $B\cap X$ is closed in X and $B\cap Y = Y$ or

2 $B\cap X =\phi$ and $B\cap Y$ is closed in Y

2.2 Properties

The join operator has the following properties for all smallest neighborhood spaces X, Y and Z

1 $X=\phi \vee X= X\vee \phi$

2 $(X \vee Y) \vee Z = X\vee (Y \vee Z)$

3 $(X \vee Y)'= Y' \vee X'$

2.3 Theorem

Let X and Y be smallest neighborhood spaces

1 $X\vee Y$ is T_0 if and only if X and Y are T_0.

2 $X \vee Y$ is compact if and only if Y is compact.

3 If $X\neq\phi$, $Y\neq\phi$ then $X \vee Y$ is connected.

Proof

1 Let X be open in $X\vee Y$. If $x \in X$ and $y\in Y$ then X is an open set containing x but not y. It follows that $X \vee Y$ fail to be To only for a pair of points in X or a pair of points in Y. Hence $X\vee Y$ is T_0 if and only if X and Y are T_0.

2 Assume that Y is not compact and $\{Ai\}i\in I$ is an open cover of Y without a finite subcover. Let $Bi=Ai\cup X$ for each $i\in I$. Then $\{Bi\}_{i\in I}$ is an open cover of $X \vee Y$ without a finite subcover. So if $X \vee Y$ is compact then Y is compact.

Conversely suppose Y is compact and $\{Bi\}^{n}_{i\in I}$ be an open cover of $X\vee Y$. It induces an open cover of Y with elements $B_i\cap Y$. But this cover has a finite sub cover $\{Bi\cap Y\}^{n}_{i=1}$, since Y is compact. So $\{B_i\}^{n}_{i=1}$ is a finite subcover of $X \vee Y$ since any B_i where $B_i\cap Y \neq \phi$ covers X.

3 Assume that $x \in X$ and $y\in Y$. Since $x \in N(y)$, x and y are adjacent. If $a, b \in X$ then $\{a, y, b\}$ is a connected set for the same reason. In the same way $\{c, x, d\}$ is connected if $c, d \in Y$.

All the above properties except (3) are true for general topological spaces not only for smallest neighborhood spaces.

3 Extended Khalimsky line

Let $[-\infty,\infty]_Z$ be the set obtained by adjoining two elements $-\infty$ and $+\infty$to the set Z. We can extend the ordering of Z by putting $-\infty < m< +\infty$ for all $m\in Z$. We can endow $[-\infty,\infty]_Z$ with a suitable topology so that it is a compactification of Z. In general topology, the spaces involved in a compactification must satisfy the Hausdorff separation axion. But this is not the case with Khalimsky line. A compactification of a smallest neighborhood X is a compact topological space Y such that X is homeomorphic to a subset of Y and X is dense in Y. The family of sets $\{[2m+1], [2m, 2m \pm 1], [2m+1, +\infty]_Z, [-\infty,2m+1]_Z, m\in Z\}$is a basis for $[-\infty,\infty]_Z$. Z as a sub subspace of $[-\infty,\infty]_Z$ is the ordinary Khalimsky line and the closure of Z in this space is $[-\infty,\infty]_Z$ .Since $[-\infty,\infty]_Z$ has both a largest and smallest element, it constitutes complete lattice in the extended ordering of Z.

3.1 Theorem

In any compactification of Z, there exist a connected smallest neighborhood space.

Proof

Let K= Z $\cup\{\infty\}$ and B$\subset$ K open if B is contained in Z and open there or if B= K. However in this topology every point is connected to ∞ since N (∞) = K.

3.2 Theorem

There exist no compactification of Z which is also a smallest neighborhood space such that no point in Z adjacent to a point at infinity.

Proof

Let K = Z$\cup$ X be a compactification of Z where X is not empty and K $\cap$Z $\cong$ Z. Since every point in Z is separated from every point in X by the smallest neighborhood property, any set open in Z is open in K and any set closed in Z is closed in K. Hence since K is a smallest neighborhood space and Z can be covered both by closed and open sets, Z constitutes a connectivity component which is not compact ie K is not compact.

4 Conclusion

We have discussed properties of joint operator and extended Khalimsky line, in this paper.

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